III Semester M.Sc. in Mathematics Examination, September/October 2020

TOPOLOGY

Time: 3 Hours **Max. Marks: 80**

Instructions: Answer any Five full questions. All questions carry equal marks.

1. a) Define Base for the topology. Let (X, τ) be a topological space. Prove that a subfamily $\mathcal{B}$ of τ is a base for τ if and only if $U \in \tau$ and $x \in U$ implies there is a B in $\mathcal{B}$ such that $x \in B \subseteq U$.

b) Define limit point with usual notations, prove that

i. $d(\emptyset) = \emptyset$

ii. $A \subseteq B \Longrightarrow d(A) \subseteq d(B)$

iii. $d(A \cup B) = d(A) \cup d(B)$ **(8+8)**

2. a) Define a continuous map. Prove that a map $f: (X, \tau) \rightarrow (Y, \upsilon)$ is continuous at $x \in X$ if and only if for every neighbourhood V of $f(x)$, $f^{-1}(x)$ is a neighbourhood of x.

b) Prove that a bijective map $f: (X, \tau) \rightarrow (Y, \upsilon)$ is a homeomorphism if and only if $f(\bar{A}) = \overline{f(A)}$, for all $A \subseteq X$. **(8+8)**

3. a) Prove that a subset A of $\mathbb{R}$ is connected if and only if A is an interval.

b) Prove that the components of a space X are connected disjoint subsets of X whose union is X, such that each connected subset of X intersects only one of them. (**10+6)**

4. a) Define a compact space. Prove that a continuous image of a compact space is compact.

b) Prove that every closed subset of a sequentially compact space is sequentially compact. **(8+8)**

5. a) Prove that second countability is both a topological and hereditary property.

b) Prove that a metric space is normal and hence T_4. **(8+8)**

6. a) If a normal space is regular then prove that it is completely regular.

b) State and prove Urysohn's Metrization theorem. **(4+12)**

7. State and prove Tychnoff theorem. **(16)**

8. a) If X is a regular paracompact space and Y is a regular σ - compact space then prove that $X \times Y$ is paracompact.

b) Prove that every paracompact space is normal. **(8+8)**

Math 3.2

III Semester M.Sc. in Mathematics Examination, September/October 2020

MEASURE AND INTEGRATION

Time: 3 Hours **Max. Marks: 80**

Instructions: Answer any Five full questions. All questions carry equal marks.

1. **a)** Let A be an algebra of X and $\{A_i\}$ a sequence of sets in A. Then prove that there is a sequence $\{B_i\}$ of sets in A such that $B_n \cap B_m = \emptyset$ for $n \neq m$ and $\bigcup_{i=1}^{\infty} B_i = \bigcup_{i=1}^{\infty} A_i$.
b) Let A be any set of real numbers. If $E_1, E_2, \ldots, E_n$ are disjoint Lebesgue measurable sets. Then prove that, $m^*(A \cap [\bigcup_{i=1}^{n} E_i]) = \sum_{i=1}^{n} m^*(A \cap E_i)$. **(10+6)**

2. **a)** Let $\{E_n\}$ be an infinite decreasing sequence of Lebesgue measurable sets, that is, a sequence with $E_{n+1} \subset E_n$, $n = 1,2,3, \ldots$ Let mE_1 be finite. Then prove that, $m(\bigcap_{i=1}^{\infty} E_i) = \lim_{n\to\infty} \text{m}E_n$.
b) Let $E \subset [0,1)$ be a Lebesgue measurable set. Then for each $y \in [0,1)$, Prove that the set $E + y$ is Lebesgue measurable and $m(E + y) = mE$. **(8+8)**

3. **a)** If f is a bounded function defined on a measurable set E with $mE < \infty$. Then prove that f is measurable if and only if, $inf_{\psi \geq f} \int \psi = sup_{\phi \leq f} \int \phi$.
b) State and prove bounded convergence theorem for a measurable function. **(8+8)**

4. **a)** Let f and g be non-negative measurable functions. If f is integrable over E and $g(x) < f(x)$ on E. Then prove that g is also integrable on E and $\int_E (f-g) = \int_E f - \int_E g$.
b) State and prove Lebesgue convergence theorem for integrable functions. **(6+10)**

5. State and prove Vitali Lemma. **(16)**

6. **a)** If f is a function of bounded variation on $[a, b]$ then prove that the following:
i. $P - N = f(b) - f(a)$
ii. $P + N = T$, where P, N, T are the positive, negative and total variation of f over $[a, b]$.
b) If f is absolutely continuous on $[a, b]$ and $f'(x) = 0$ almost everywhere then prove that f is constant. **(8+8)**

7. **a)** A function F is an indefinite integral if and only if it is absolutely continuous.
b) Let (X, B, μ) be a measure space. If $E_i \in B$ for $i = 1,2,3, \ldots, \mu E_1 < \infty$ and $E_i \supset E_{i+1}$, $i = 1,2,3, \ldots$ then prove that $\mu[\bigcap_{i=1}^{\infty} E_i] = \lim_{n\to\infty} \mu E_n$. **(8+8)**

8. **a)** State and prove Lebesgue Decomposition Theorem.
b) If $A \subset \mathcal{A}$ and if $\{A_i\}$ is any sequence of sets in $\mathcal{A}$ such that $A \subset \bigcup_{i=1}^{\infty} A_i$ then prove that, $\mu A \leq \sum_{i=1}^{\infty} \mu A_i$. **(8+8)**

Math 3.3

III Semester M.Sc. in Mathematics Examination, September/October 2020
FUNCTIONAL ANALYSIS

Time: 3 Hours **Max. Marks: 80**

Instructions: Answer any Five full questions. All questions carry equal marks.

1. **a)** Define limit point of a set, closure of a set and dense set. If A is a finite in a metric space (X, d), then show that A', complement of A is open set.
b) If $\bar{A}$ denotes the closure of a subset A of a metric space (X, d), then show that $\bar{A} = \{x \in X : d(x, A) = 0\}$. **(10+6)**

2. **a)** If (X, d) is a complete metric space and (Y, d) be a subspace of (X, d). Then show that Y is complete if and only if Y is closed.
b) If (X, d) is a complete metric space and if $T: X \to X$ is a contraction mapping then show that T has a unique fixed point. **(8+8)**

3. **a)** State and prove Cantor's Intersection theorem.
b) Make use of Cantor's Intersection theorem to prove Baire's category theorem. **(8+8)**

4. **a)** Prove that, every sequentially compact metric space is both complete and totally bounded.
b) If $T: N \to N'$ is a linear operator from a normed linear space N into a normed linear space N' then show that the following are equivalent.

i. T is continuous linear operator
ii. T is continuous at $x = 0$
iii. T is a bounded linear operator. **(8+8)**

5. **a)** Prove that a normed linear space X is complete if and only if every absolutely convergent series in X is convergent.
b) State Hahn- Banach theorem for a normed linear space. Prove the theorem by making use of Hahn-Banach theorem for a complex linear space. **(8+8)**

6. **a)** If N is a normed linear space and x_0 is a non-zero vector in N, then show that there exists a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$.
b) State and prove open mapping theorem. **(8+8)**

7. **a)** State and prove uniform bounded principle.
b) If B be a Banach space and N is a normed linear space. Let $\{T_n\}$ is a sequence in $\mathbb{B}(B, N)$ such that $\lim_{n\to\infty} T_n(x) = T(x)$ exists for all $x \in B$, then show that $T \in \mathbb{B}(B, N)$. **(8+8)**

8. **a)** Prove the following two properties of Hilbert space H:

i. $\|x + y\|^2 + \|x - y\|^2 = 2(\|x\|^2 + \|y\|^2)$
ii. $4(x, y) = \|x + y\|^2 - \|x - y\|^2 + i\|x + iy\|^2 - i\|x - iy\|^2$.

b) If T is an operator on H for which $(T(x), x) = 0$ for all x, then show that $T = 0$.
c) If $\{e_i\}$ is an orthonormal set in a Hilbert space H and if $x \in H$, then show that $\sum|(x, e_i)|^2 \leq \|x\|^2$.
(5+5+6)

Math 3.4

III Semester M.Sc. in Mathematics Examination, September/October 2020

MATHEMATICAL MODELING

Time: 3 Hours **Max. Marks: 80**

Instructions: Answer any Five full questions. All questions carry equal marks.

1. **a)** Explain the techniques of Mathematical modeling.
b) Explain any four characteristics of Mathematical modeling.
c) Velocity of a moving particle is given by $v = t^2 - \frac{3x}{t}$; where v is velocity, x displacement and t is time. Find an expression for the displacement of the particle at any time t. **(6+6+4)**

2. **a)** Consider the differential equation $\frac{dx}{dt} = 2\cos(\pi x)$, find all equilibria and determine the stability of those equilibria.
b) Explain linear population growth model. Find the expression for doubling / half- life period.
(8+8)

3. **a)** Explain the construction of spring and dashpot system.
b) Describe a model for the detection of diabetes. **(8+8)**

4. **a)** Find the expression for velocity and acceleration vectors along radial and transverse direction.
b) Show that the force required to make a particle of mass ‘ m’ move in a circular orbit of radius with velocity ‘v’ is $\frac{mv^2}{a}$ directed towards the centre. **(8+8)**

5. **a)** Show that velocity profile for irrotational flow of a fluid is a harmonic function.
b) Describe a partial differential equation model for birth-death-immigration process. **(8+8)**

6. **a)** Describe a model for Glacier flow.
b) Derive Burger’s equation for the convection diffusion process. **(8+8)**

7. **a)** What are the sources of air pollution? Explain.
b) Distinguish between primary and secondary air pollutants.
c) List out any two major toxic metals and write their effects. **(6+6+4)**

8. **a)** Write a note on Suspended Particulate Matter (SPM) and mention some of its effects on health, climate and vegetation.
b) Give mathematical formulation of SPM in Taylor Dispersion Model. **(6+10)**

III Semester M.Sc. in Mathematics Examination, September/October 2020

COMPUTER PROGRAMMING

Time: 3 Hours **Max. Marks: 80**

Instructions: Answer any Five full questions. All questions carry equal marks.

1. **a)** What are the differences between RAM and ROM?
b) Explain primary memory its properties and its types?
c) Describe the evolution and roles of Assembly language and Assembler in computer programming.
(4+6+6)

2. **a)** Explain the method for developing the algorithm. Illustrate the method to write an algorithm to compute the area of a circle.
b) Explain the memory architecture of parallel computer.
c) Explain the types of networks along with examples. **(7+5+4)**

3. **a)** Describe in detail the basic structure of C- programing.
b) Write a C-program to print your name on the computer screen.
c) What is the storage class and explain the different storage class and its specifications. **(7+2+7)**

4. **a)** Describe briefly an increment and decrement operators.
b) Explain briefly Operator precedence along with example.
c) Explain the following along with example.
(i) For loop **(ii)** While loop **(4+4+8)**

5. **a)** Explain declaration and initialization of a two dimension array with an example.
b) Write an algorithm and C-programming for binary searching.
c) Explain the techniques of selection sort? Write a program to arrange N number in the ascending order using source code of simple selection sort method. **(5+5+6)**

6. **a)** Explain the types of function which support in C.
b) Write a C program to find the factorial of a given number using recursive function.
c) What is a pointer and how to understand the pointer. Explain the pointer declaration along with examples. **(7+3+6)**

7. **a)** Write an algorithm and C program for an entered number is prime or not.
b) What is a bisection method? Write an algorithm and C program for the bisection method.
(7+9)

8. **a)** Write an algorithm and C program for the Gauss Elimination Method.
b) Write C program for the Trapezoidal rule. Illustrate with an example. **(9+7)**